

Effects of Reliability Measures on Market Share

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Abstract—Although unavailability and failure frequency are both important reliability measures for designing telecommunications networks, having two different reliability measures causes difficulties when their evaluation results contradict each other. To overcome these difficulties, I propose an approach to determining a single reliability measure to be used. I focus on market share, which is a very important management index, and study the effect of each reliability measure on this index. If we find that the effect of either reliability measure is small enough to be ignored, then we choose the other one as the single reliability measure. I give a statistical analysis procedure for this study and demonstrate test results that show failure frequency to be promising as the single reliability measure.

Index Terms— Reliability, Availability, Failure frequency, Market share

I. INTRODUCTION

A lot of research has been done on trying to find a method of achieving high reliability in telecommunications networks. Many papers^{[1]-[5]} emphasize that the following steps achieve high reliability.

- Step 1: Define the reliability measure.
- Step 2: Enumerate alternative design plans for network topology and back up.
- Step 3: Evaluate the reliability of each design plan using the reliability measure defined in Step 1.
- Step 4: Select a sufficiently reliable design plan.
- Step 5: Implement it.
- Step 6: Feed the problems found in implementation process back to the next activity.

While the above steps are all important, almost all previous studies have shown little interest in Step 1. They chose unavailability in this step and never considered other reliability measures. However, it has been claimed that failure frequency is also a very important reliability measure [6]-[8]. Even if we find a design plan that is reliable in terms of unavailability evaluation, we often find that it is unreliable when failure frequency is evaluated (and vice versa). Therefore, both reliability measures were considered to be important for designing telecommunications networks [6]-[8].

Their claims are reasonable, but we must say that having two reliability measures causes difficulties because it is not clear how to design networks when we find contradictions between them.

In this paper, I propose the first step toward overcoming

these difficulties by determining the single reliability measure to be used, by selecting the best one from two existing reliability measures. I focus on market share, which is a very important management index, and study the effect of each reliability measure on the market share. If we find that the effect of either of the reliability measures is small enough to be ignored, then we can choose the other one as the single measure. I give a statistical analysis procedure for this study and demonstrate test results that show that failure frequency is promising as the single measure.

II. PREPARATION

This section explains the words and mathematical definitions used in this paper.

A. Preparation for describing reliability measures

- uptime: Time from ‘recovery of service’ to next ‘failure occurrence again’
- downtime: Time from ‘failure occurrence’ to ‘recovery’
- MUT: Mean uptime
- MDT: Mean downtime

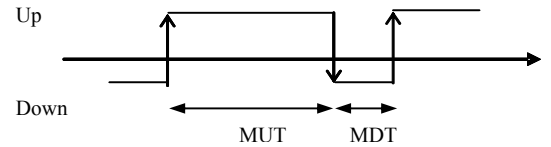


Fig. 1. MUT & MDT.

- unavailability: $MDT/(MUT+MDT)$
- failure frequency: $1/(MUT+MDT)$

B. Preparation for describing the indices of market share

Suppose that we have companies A_1, A_2, \dots, A_n in competing to supply the same service.

- t, t_0 : Dates used in this study, where $t \geq t_0$ (I call t_0 the key date.)
- $N(A_i, t)$: Number of users of Company A_i on date t
- $\alpha(t)$: Index representing the size of the market on date t defined as

$$\alpha(t) = N(A_1, t) + N(A_2, t) + \dots + N(A_n, t)$$
- $S(A_i, t)$: Index representing the market share of Company A_i defined as

$$S(A_i, t) = N(A_i, t) / \alpha(t)$$

$r(A_i, t, t_0)$: Index representing how much the market share of Company A_i fluctuates (increase or decrease) after key date t_0 , defined by

$$r(A_i, t, t_0) = \{S(A_i, t) - S(A_i, t_0)\} / S(A_i, t_0)$$

I call $r(A_i, t, t_0)$ the ‘market share fluctuation rate’ or ‘fluctuation rate of market share of Company A_i ’.

- T : Number of days elapsed since the key date
 $T = t - t_0$
- T_1 : Observation period for computing the market share fluctuation rate (I compute $r(A_i, t, t_0)$ when $0 < t \leq T_0$.)
- T_2 : Threshold value for defining many failures in a short period (I define ‘many failures in a short period’ as two failures in period T_2 .)
- d : Threshold value for defining long downtime failure (I define a ‘long downtime failure’ as a failure whose downtime is more than d . (See Subsection *D* of Section IV.)

III. ASSUMPTIONS ABOUT DATA FOR ANALYSIS

I assume that we can obtain the failure data of each company for a period of a certain number of years (denoted X years). These data include the date of occurrence and the length of downtime for each failure. I also assume that we can obtain the market share data of each company in the same period. Such data are not so difficult to obtain from open data such as white papers and economical journals. (see Refs. [9]-[10])

IV. ANALYSIS METHOD

For the reliability design process, we must determine the appropriate reliability measure carefully considering whether it really reflects users’ activities in selecting services. If users select the services of a rival company because it has fewer failures, then we should select a reliability measure that expresses how frequently failures occur. If users select the services of a rival company because it performs quick repairs, then our reliability measure should express the speed of repair.

Now, in this section, I present an analysis method for determining the reliability measure by using data satisfying the assumptions of Section III. First, I clarify the index expressing the tendency of user actions for selecting service supplying company. Second, I explain the difficulties about the analysis. Third, I explain the framework of my analysis for overcoming the difficulties. Finally, I explain the detailed procedure of the analysis.

A. Index expressing tendency of user actions

I use the market share fluctuation rate as the index expressing the tendency of user actions for selecting a service-supplying company because these user actions surely reflect this index.

B. Difficulties of analysis

If we find that one of the reliability measures (unavailability or failure frequency) affects the market share

but the other measure has a small enough effect to be ignored, then we can choose the single measure to be used. Therefore, my aim is to analyze the effect of each reliability measure on market share.

There are many statistical analysis methods that can be applied for this analysis. However, appropriate method must be carefully chosen to avoid misleading results. For example, it might seem reasonable to compute two correlation coefficients of ‘unavailability vs. market share’ and ‘failure frequency vs. market share’ to solve our task. However, we cannot say that these correlation coefficients analysis are appropriate for our task for the following reasons.

Reason 1: Even if the results of this analysis show that both correlation coefficients are near to 1, we cannot conclude that both measures affect the market share because unavailability & failure frequency might have a mutually correlation for the data.

(If there is correlation between two measures, a correlation coefficient close to 1 might be caused by the other correlation coefficient value.)

Reason 2: This correlation coefficient analysis might give different results depending on how unavailability and failure frequency are estimated from data. For example, failure frequencies estimated as the average number of failure occurring in a one-month period, one-year period, and longer periods will lead to different results.

Reason 3: There might be a delay in the effect being reflected in the market share. A correlation coefficient analysis is not sufficient to clarify such a delay.

To avoid misleading results, we should perform the analysis taking into consideration how the failure occurrences affect the fluctuation of market share in actual situations.

C. Framework of analysis method

I propose an analysis framework for overcoming the above-mentioned difficulties.

- (1) We should consider the mechanism of market share fluctuation. That is, first a failure occurs, then it causes user actions for selecting a service-supplying company, and finally the market share fluctuation decreases.
- (2) Item (1) says that the decrease in market share occurs after a failure. Therefore, we should focus on the fluctuation in market share after each failure.
- (3) We should also consider that there are some patterns of failures, such as many failures in a short period and failures with long downtimes.
- (4) Users never select the service supplying company by ob-

serving computed results for reliability measures but select by observing the patterns of failures that occur and that they experience.

- (5) That is, we must say that the ‘effect of the reliability measure on market share’ are the results caused by the following user actions.

- Action 1: Users select rival companies if many failures occur in a short period.
Action 2: Users select rival companies if long-downtime failures occur.

If the majority of users show the tendency of Action 1 then we can say that the failure frequency affects the market share. If the majority of users show the tendencies of both Action 1 & 2, then we can say that unavailability affects the market share. (These are consequences of the definitions of the reliability measures.)

- (6) Consequently, the analysis for solving our task is to clarify whether the majority of users have either or both of these tendencies.

(I must say that this framework is not a complete solution for the analysis. For example, one should analyze not only failures but also other various aspects such as ‘competitive prices’. However, I believe that the above framework is more advanced than the correlation coefficient analysis described in B. The analysis method will make progress step by step.)

The analysis procedure in the above framework is explained in detail in the next Subsection.

D. analysis procedure

The analysis procedure consists of three subprocedures.

- Subprocedure 1. Preparatory analysis
Subprocedure 2. Analysis for failure frequency
Subprocedure 3. Analysis for unavailability

Subprocedure 1. Preparatory analysis

As the preparatory analysis, we construct a *BASIC DIAGRAM* representing the market share fluctuation after each failure occurrence, as illustrated in Fig. 2.

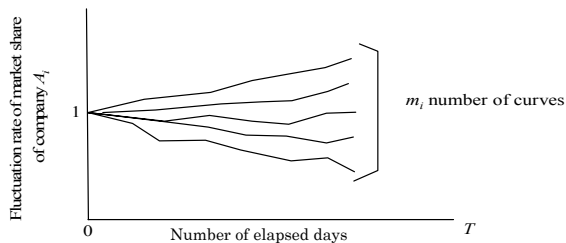


Fig. 2. *BASIC DIAGRAM*.

(m_i is the number of failures of company A_i in the data)

This diagram is drawn by the following steps.

- Step 1. We specify key date t_0 as the date of a failure occurrence of Company A_i .
Step 2. We compute $r(A_i, t_0 + T, t_0)$, where $T \leq T_1$.
Step 3. We draw $r(A_i, t_0 + T, t_0)$ on a graph representing the fluctuation rate of the market share of Company A_i after each key date.
Step 4. We repeat Steps 1, 2, & 3 for every failure occurrence of Company A_i .

After creating the *BASIC DIAGRAM*, we analyze separately whether unavailability and failure frequency affect the market share.

Subprocedure 2. Analysis for failure frequency

Due to the definition of failure frequency, a large value of it implies that many failures occur in a short period. Therefore, “Effect of failure frequency on market share” is equivalent to “Users tend to select rival companies if they experience a pattern of many failures in a short period.” Accordingly, the following steps clarify the effect of failure frequency on market share.

- Step 1. We create a new diagram by choosing the curves produced by the patterns of many failures in a short period from the *BASIC DIAGRAM* by the following substeps.
Substep 1-1. Search for a case of two failure occurrences (denoted by failures F_1 & F_2) such that the interval between F_1 & F_2 is less than T_2 .
Substep 1-2. Choose the curve of the later of the two failures F_1 & F_2 .
Substep 1-3. Repeat Substeps 1-1 & 1-2 until we succeed in choosing every pair of failures in Substep 1-1.
Step 2. On this new diagram, we statistically verify the declines of the curves by using the procedure given in the APPENDIX.

Subprocedure 3. Analysis for unavailability

From the definitions of unavailability & failure frequency, we can derive the following equation.

$$\text{Unavailability} = \text{failure frequency} \times \text{MDT}$$

This equation implies that unavailability is a combined measure of failure frequency & MDT. Remember that we have already analyzed the effect of failure frequency on market share in Subprocedure 2. Therefore, our task for unavailability is reduced to analyzing whether or not MDT affects the market share.

MDT is defined as the mean downtime. Therefore, “The effect of MDT on market share” is equivalent to “Users tend to select rival companies if they experience a pattern of long-downtime failures.” Based on this equality, the following steps clarify the effect of availability on market share.

Step 1. Create a new diagram by choosing the curves of the patterns of long-downtime failures from the *BASIC DIAGRAM* by the following Substeps.

Substep 1-1. Search for a failure (denoted by F) whose downtime is more than d .

Substep 1-2. Choose the curve produced by F .

Substep 1-3. Repeat Substeps 1 & 2 until we find every short-downtime failure in Substep 1.

Step 2: On this new diagram, we statistically verify the declines of the curves by using the procedure in the APPENDIX.

V. TEST

This section describes how the analysis was tested with real data. Subsection *A* explains the data used for the test. Subsection *B* shows the specific values of parameters of T_1 , T_2 , and d used in the test. Subsection *C* shows the results.

A. Explanation of data used for test

The data used for this test were as for two companies A_1 & A_2 for 5 years, where these companies are rivals for the same service. The data were collected by a public organization.

B. Specifications of parameters

We specify the values of T_1 , T_2 , & d in the analysis for this test as $T_1 = 3$ months, $T_2 = 2$ months, and $d = 4$ hours.

' $T_1 = 3$ months' is reasonable because we find that curves show irregular fluctuations in some diagrams if $T > T_3$, which causes difficulties in applying statistic analysis.

' $T_2 = 2$ months' & ' $d = 4$ hours' are reasonable because if we use other values, the number of curves in some diagrams becomes too small, such as zero, one, or two, which causes difficulties in applying statistical analysis.

C. Analysis results

1) Results for Company A_1

The *BASIC DIAGRAM* for Company A_1 , showing market share fluctuation rate after each single failure, is shown in Fig. 3. (For Figs. 3-5, we ignore a failure of Company A_1 if its occurrence date is in the irregular weeks for which we have too many almost simultaneous failures of Company A_2 : specifically 4 weeks having 4 failures of Company A_2 .)

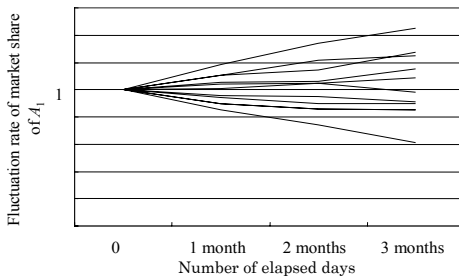


Fig. 3 *BASIC DIAGRAM* (Company A_1).

The diagram obtained by choosing failure patterns of many failures in a short period is shown in Fig. 4.

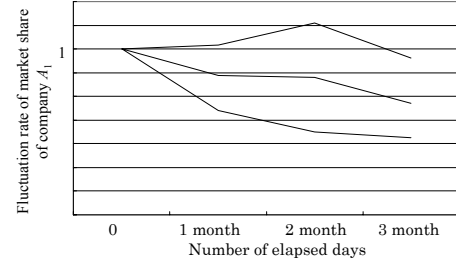


Fig. 4. Diagram for many failures in short period (Company A_1).

The diagram obtained by choosing failure patterns of long downtime is shown in Fig. 5.

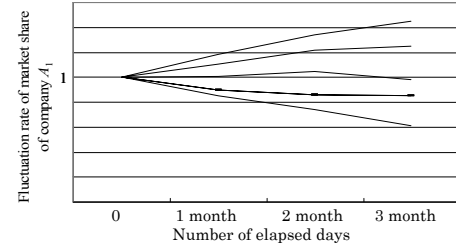


Fig. 5. Diagram for long downtime failures (Company A_1).

Table 1 shows the results of statistically verifying the effect on market share by the procedure in APPENDIX for each diagram.

Table 1. Verification results (Company A_1).

Patterns of failures	Result
The failures of <i>BASICDIAGRAM</i> (Fig.3)	NOT VERIFIED
Many failure in short period (Fig.4)	VERIFIED
Long downtime failures (Fig.5)	NOT VERIFIED

(In the table, 'VERIFIED' indicates that the procedure in APPENDIX verified the effect on market share. 'NOT VERIFIED' indicates that it did not verify the effect.)

These figures & the table show the following.

- Figure 3 and the first line of Table 1 show that there is no evidence for failures causing a decrease in market share if we do not consider failure patterns.
- Figure 4 and the second line of Table 1 show that the decreases in market share of Company A_1 were caused by the patterns of many failures in a short period.
- Figure 5 and the third line of Table 1 show that there is no evidence for patterns of short-downtime failures causing decreases in market share.

ii) Results for Company A_2

The results for Company A_2 are shown in Figs.6-8 and Table 2 in the same manner as in i).

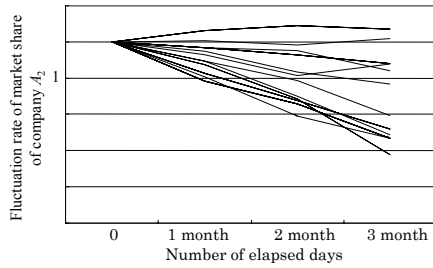


Fig. 6. *BASIC DIAGRAM* (Company A_2).

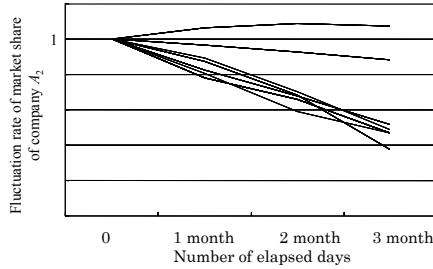


Fig. 7. Diagram for many failures in a short period (Company A_2).

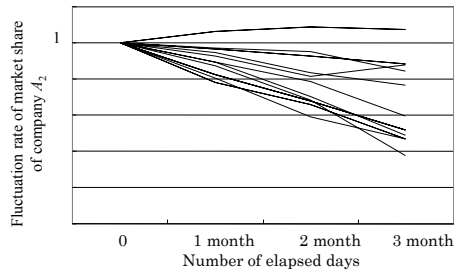


Fig. 8. Diagram for long downtime failures (Company A_2).

Table 2. Verification results (Company A_2).

Patterns of failures	Result
Failures of <i>BASICDIAGRAM</i> (Fig. 6)	VERIFIED
Many failure in a short period (Fig. 7)	VERIFIED
Long-downtime failures (Fig. 8)	NOT VERIFIED

NOTE: Figures 6-8 show that almost all curves showed declines. However, these declines did not always indicate the effect causing the decrease in market share because the data of Company A_2 also had a clear tendency of declining curves even when we chose key dates with no consideration of failure. (See Fig. AP-2. in APPENDIX.) We must compare each diagram with the case in which the key dates are chosen by a policy unrelated to failures. The procedure in APPENDIX did this comparison.

These figures and the table show the following.

- Figure 6 and the first line of Table 3 show that there were decreases in market share when Company A_2 caused failures.
- Figure 7 and the second line of Table 2 show that there were also decreases in market share when Company A_2 caused patterns of many failures in a short period.
- Figure 8 and the third line of Table 2 show that there is no evidence of patterns of long-downtime failures causing decreases in market share.

(I confess that this section shows very limited test results and there could be counterarguments against the above statements. Through proofs are topics for future work.)

VI. DISSCUSSION

The followings findings were obtained from the results given in section V.

1. The results for Company A_1 indicate that a pattern of many failures in a short period causes a decrease in market share.
2. The results for Company A_2 never contradict the above indication.
3. Results for both Companies A_1 & A_2 never indicate that patterns of long-downtime failures cause a decrease in market share.

From the logic of (5) of Subsection C in Section IV, the above three items lead to failure frequency being promising as the single measure to use.

VII. CONCLUSION

This paper studied the effects of unavailability and failure frequency on market share to enable us to determine the single reliability measure to be utilized for users not to select rival companies. Test data results show that failure frequency is promising as the single reliability measure. Future works includes:

- Applying the same analysis to data collected for many years
- Applying the same analysis to data in other fields of services
- Constructing a reliability design method based on failure frequency

APPENDIX

PROCEDURE FOR VERFYING DECREASES

This APPENDIX shows the procedure for statistically verifying the effect on market share in each diagram. The difficulty of verification arises from the fact that even if we find almost all curves in the diagram of A_1 declining, we still cannot say that we have found the effect on market share, because the market share might have decreased for reasons unrelated to failures. Therefore, verifying 'the effect on failures on market

share' implies 'verifying the declines of curves that are stronger than those of curves whose key dates were chosen without consideration of failures'. On the basis of this idea, the verification procedure is given below. (Remember that 'X years' denotes the period for collecting data, as described in Section III. We use D_i to denote the diagram in which we want to verify declines.)

Step 1. We create a new diagram in the same way as for the *BASIC DIAGRAM* except for the policy of choosing key dates. The new policy is that we choose key dates t_1', t_2', \dots, t_K' without considering failures and impartially over X years. We call this new diagram the *REFERENCE DIAGRAM*

Step 2. We compare diagram D_i and the *REFERENCE DIAGRAM*.

Step 3. If we verify stronger declines of curves in D_i than in *REFERENCE DIAGRAM* by the following way, then we have found the effect on market share.

Verification is

We assume the following null hypothesis and alternative hypothesis and apply the standard statistical method as shown in [11] to verify the declines.

Null hypothesis: $\theta = \theta_0$

Alternative hypothesis: $\theta < \theta_0$

In the above, θ and θ_0 are defined as follows.
(See Subsection D of Section IV for T_1 .)

θ : Average of the values of market share fluctuation rates in D_i for $T = T_1$

θ_0 : Average of the values of market share fluctuation rates in *REFERENCE DIAGRAM* for $T = T_1$.

The first date of every month for X years is chosen as each key dates t_1', t_2', \dots, t_K' for the case in Section V. Figures AP-1 and AP-2 show *REFERENCE DIAGRAMs* for this case

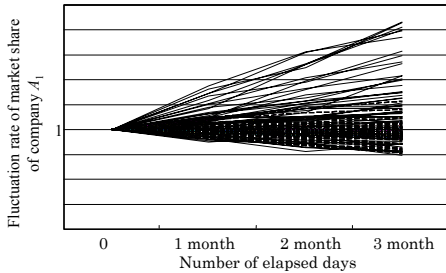


Fig. AP-1. *REFERENCE DIAGRAM* (Company A_1).

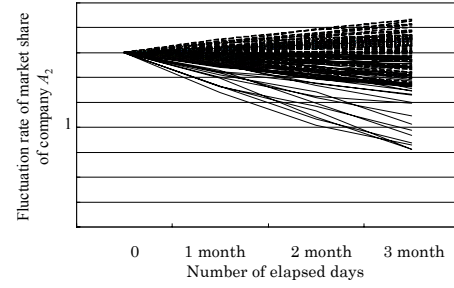


Fig. AP-2. *REFERENCE DIAGRAM* (Company A_2).

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